Vickrey Auction with Single Duplicate Approximates Optimal Revenue

Hu Fu
UBC

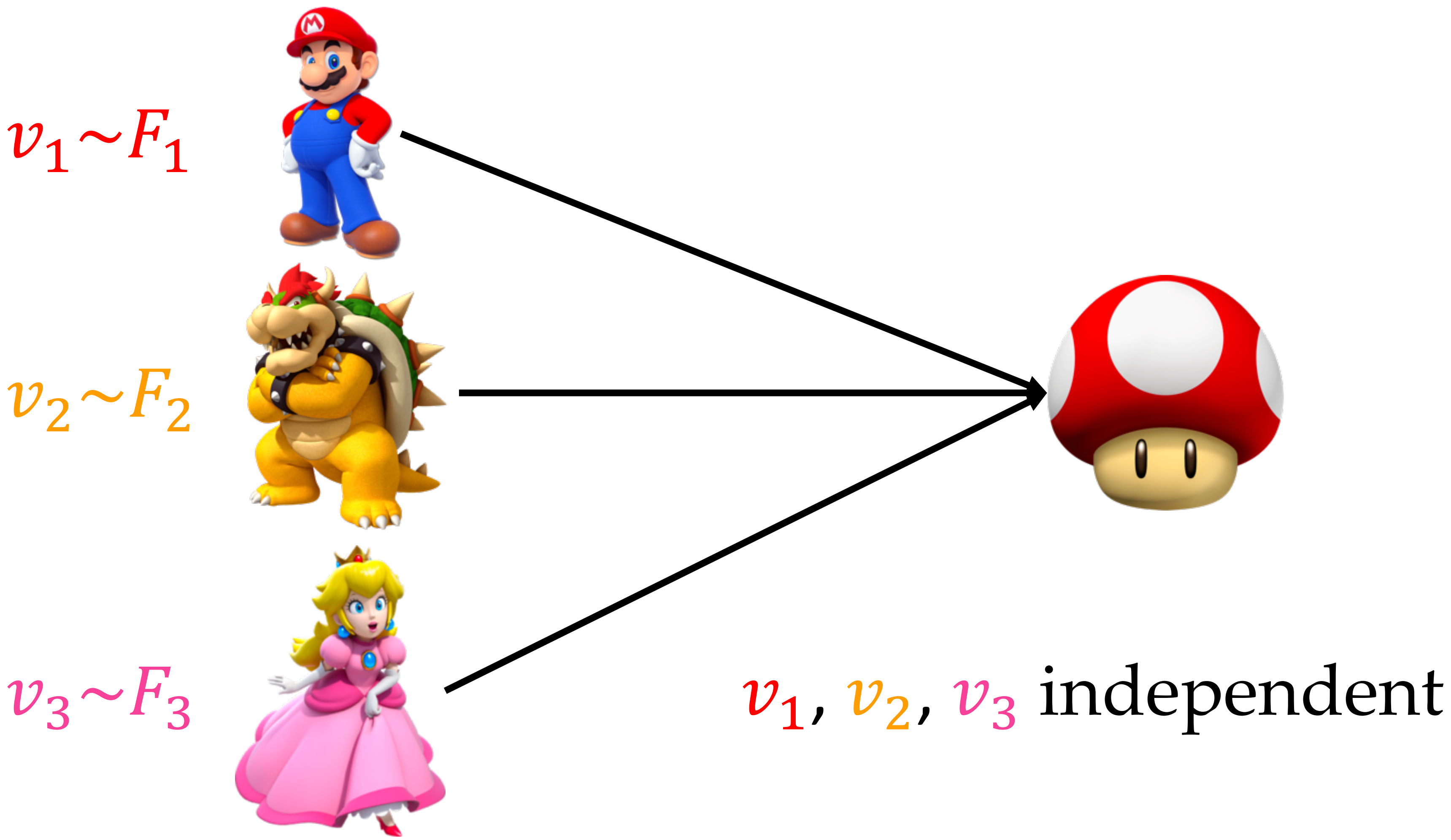
Sikander Randhawa
UBC

Chris Liaw (UBC)

EC '19, June 2019

Setting

- n bidders, single item



Bulow and Klemperer's Theorem

Second price (Vickrey) auction

✓ Simple and prior-free
✓ Efficient allocation
✗ May have poor revenue

Revenue-optimal auction

✗ Complex auction
✗ Requires prior knowledge
✓ Maximizes revenue

William Vickrey

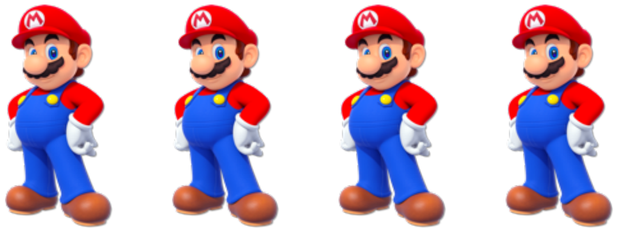
Roger Myerson

Bulow and Klemperer's Theorem

Second price (Vickrey) auction
- ✓ Simple and prior-free
- ✓ Efficient allocation
- ✘ May have poor revenue

Revenue-optimal auction
- ✘ Complex auction
- ✘ Requires prior knowledge
- ✓ Maximizes revenue

Theorem. [Bulow, Klemperer '96] Given $\boldsymbol{n}$ **i.i.d.** bidders, the second price auction with **one** additional bidder, from the same distribution, yields at least as much revenue as the optimal auction with the original $\boldsymbol{n}$ bidders.
[assuming value distributions are "regular"]



Second price auction $\geq$ **Optimal auction**

Bulow and Klemperer's Theorem

Second price (Vickrey) auction

✓ Simple and prior-free
✓ Efficient allocation
✗ May have poor revenue

Revenue-optimal auction

✗ Complex auction
✗ Requires prior knowledge
✓ Maximizes revenue

Theorem. [Bulow, Klemperer '96] Given $\boldsymbol{n}$ **i.i.d.** bidders, the second price auction with **one** additional bidder, from the same distribution, yields at least as much revenue as the optimal auction with the original $\boldsymbol{n}$ bidders.
[assuming value distributions are "regular"]

Q: Is a similar result true when distributions are not identical?

It does **not** work to choose an arbitrary bidder and recruit a copy.

E.g., what if only **Mario** has a high value for mushroom?

A non-i.i.d. version of BK

Theorem. [Hartline, Roughgarden '09] Given ***n*** **independent** bidders, the second price auction with ***n*** additional bidders, one from each given distribution, yields at least **half** as much revenue as the optimal auction with the original ***n*** bidders. [assuming value distributions are "regular"]

Second price auction $\geq$ ½ · **Optimal auction**

A non-i.i.d. version of BK

Theorem. [Hartline, Roughgarden '09] Given n **independent** bidders, the second price auction with n additional bidders, one from each given distribution, yields at least **half** as much revenue as the optimal auction with the original n bidders. [assuming value distributions are "regular"]

Theorem. [Bulow, Klemperer '96] Given n **i.i.d.** bidders, the second price auction with **one** additional bidder, from the same distribution, yields **at least as much** revenue as the optimal auction with the original n bidders.
[assuming value distributions are "regular"]

Two key differences:

1. Recruits n bidders instead of **one**.
2. Revenue is **approximately** optimal.

Approximation is **necessary**.
Better than ¾ is impossible.

Q: How many bidders suffice for second price to be approximately optimal?
Q: Can we recruit fewer than n additional bidders? What about **one** bidder?

Main Result

Theorem. [Fu, L., Randhawa '19] Given ***n*** **independent** bidders, there **exists** **one** bidder such that the second price auction with an additional copy of that bidder yields at least ***Ω*(1) fraction** as much revenue as the optimal auction for the original ***n*** bidders [assuming value distributions are "regular"].

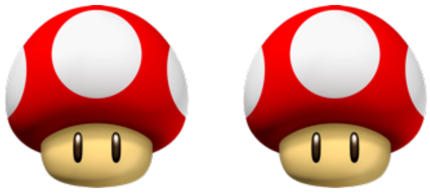
Recruit one of these.

Second price auction $\geq$ $\Omega(1)$ · **Optimal auction**

Main Result

Theorem. [Fu, L., Randhawa '19] Given ***n*** **independent** bidders, there **exists** **one** bidder such that the second price auction with an additional copy of that bidder yields at least **$\Omega(1)$ fraction** as much revenue as the optimal auction for the original ***n*** bidders [assuming value distributions are "regular"].

Remark. Techniques can be extended to show that for auctions with k identical items and n unit-demand bidders, a $(k+1)^{\text{th}}$ price auction with k additional bidders yields at least $\Omega(1)$ fraction of optimal revenue.



Main Result

Theorem. [Fu, L., Randhawa '19] Given $\boldsymbol{n}$ **independent** bidders, there **exists one** bidder such that the second price auction with an additional copy of that bidder yields at least **$\boldsymbol{\Omega(1)}$ fraction** as much revenue as the optimal auction for the original $\boldsymbol{n}$ bidders [assuming value distributions are "regular"].

Remark. Techniques can be extended to show that for auctions with k identical items and n unit-demand bidders, a $(k+1)^{\text{th}}$ price auction with k additional bidders yields at least $\Omega(1)$ fraction of optimal revenue.

Up to an **approximation**, BK theorem extends to **non-i.i.d.** setting with the **same** number of recruitments.

Additional results

Theorem. Suppose there are **2** **independent** bidders. Recruiting a copy of **each** bidder and running a second price auction yields at least ¾ **fraction** of revenue of the optimal auction with original **2** bidders.
[assuming value distributions are "regular"]

Improves on the ½-approximation and is **tight.** [Hartline, Roughgarden '09]

To prove this, we make a connection between the second-price auction with recruitments and Ronen's "lookahead auction".

En route, this gives a new proof of Hartline and Roughgarden's ½-approximation result.

Proof sketch of main result

Theorem. [Fu, L., Randhawa '19] Given n **independent** bidders, there **exists** **one** bidder such that the second price auction with an additional copy of that bidder yields at least $\Omega(1)$ **fraction** as much revenue as the optimal auction for the original n bidders [assuming value distributions are "regular"].

Theorem would be true if:

1. Second price for original bidders is approximately optimal.
2. Some bidder has high value with high probability.
   - Via a reduction to Bulow-Klemperer Theorem.

Lemma. Given n distributions, at least one of the following must be true:

1. revenue of 2^{nd} price auction is $\Omega(1) \cdot OPT$; or
2. some bidder i has value $\Omega(1) \cdot OPT$ with probability $\Omega(1)$.

[assuming "regularity"]

Rev. of optimal auction.

Overview of approach

Lemma. Given n distributions, at least one of the following is true:
1. revenue of 2nd price auction is $\Omega(1) \cdot OPT$; or
2. some bidder i has value $\Omega(1) \cdot OPT$ with probability $\Omega(1)$.

[assuming "regularity"]

Overview of approach:
1. We consider the "ex-ante relaxation", allowing us to decouple interaction amongst the bidders.
2. "Regular" distributions have nice geometric properties which we exploit on a per-bidder basis.

Ex-ante relaxation is common technique to obtain upper bounds.
[e.g. Alaei et al. '12; Alaei '14; Alaei et al. '15; Chawla, Miller '16; Feng, Hartline, Li '19]

Sketch of lemma

Lemma. Given n distributions, at least one of the following is true:

1. revenue of 2nd price auction is $\Omega(1) \cdot OPT$; or
2. some bidder i has value $\Omega(1) \cdot OPT$ with probability $\Omega(1)$.

[assuming "regularity"]

Suppose that case 2 does not hold, i.e.

$$p_i = \Pr\left[v_i \geq \frac{1}{2} \cdot OPT\right] \leq \frac{1}{2} \text{ for all } i.$$

Using properties of regularity & geometry of "revenue curves" we show that

$$\sum_i p_i \geq 1$$

Simple Fact. Suppose we flip n coins, where coin i has prob. of heads $p_i \leq \frac{1}{2}$ and $\sum_i p_i \geq 1$. Then at least two coins are heads with probability $\Omega(1)$.

Information requirements for recruitment

Theorem. [Fu, L., Randhawa '19] Given n **independent** bidders, and **assuming "mild distribution knowledge"**, there is an algorithm that decides a bidder to recruit so that the second price auction with an additional copy of that bidder yields at least $\Omega(1)$ **fraction** as much revenue as the optimal auction for the original n bidders. [assuming value distributions are "regular"]

In the paper, we give some examples of distribution knowledge which are sufficient for recruitment.

Conclusions & Open Questions

- We showed that recruiting a *single* bidder and running 2nd price yields revenue which is at least $\frac{1}{10}$ of optimal revenue.
  - Can this approximation be improved?
  - Impossible to do better than ≈ 0.694.

- If we recruit n bidders, best approximation is ½ and better than ¾ is impossible.
  - For $n = 2$, the ¾ is tight.
- Q: What is the tight approximation ratio for this setting?